Verification of Endgame Databases

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CONSTRUCTION

A chess endgame database is a map from legal positions to one of the following possible outcomes:

- a forced checkmate within n moves for the White pieces;
- a forced checkmate within n moves for the Black pieces;
- a draw (with best play neither player has a forced win).

Endgame databases are used in chess programs to improve their playing strength in positions with few pieces, and are constructed by explicit enumeration of the game graph. A symbolic version of this construction method can be implemented in the theorem prover by using BDDs to represent sets of positions. A resulting theorem

$$\vdash_{\mathsf{HOL+BDD}} \phi[B_1,\ldots,B_k] \in \mathsf{win1\_by\ chess}\ n \mapsto \Delta_{B_1,\ldots,B_k}$$

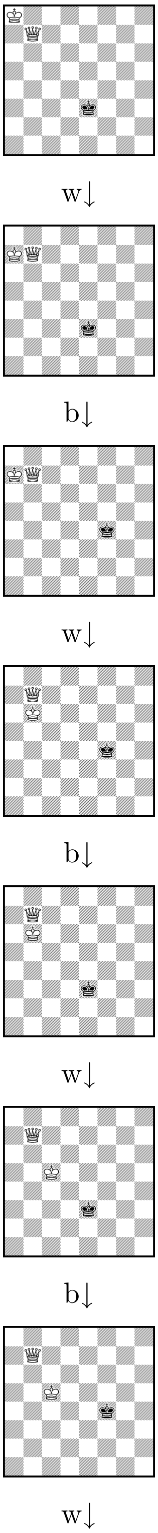
reads as: for every boolean assignment $B_i \mapsto b_i$, the chess position with boolean encoding $[b_1,\ldots,b_k]$ is a forced win within n moves for White if and only if the BDD $\Delta_{b_1,\ldots,b_k}$ evaluates to true. The BDD Δ represents a set of winning positions, and ϕ is an abstraction function that links the boolean encoding to a natural formalization of the laws of chess in higher order logic. The construction stops when the winning sets converge: the overall algorithm can be seen as a tailored instance of symbolic model checking, implemented in the HOL4 theorem prover.

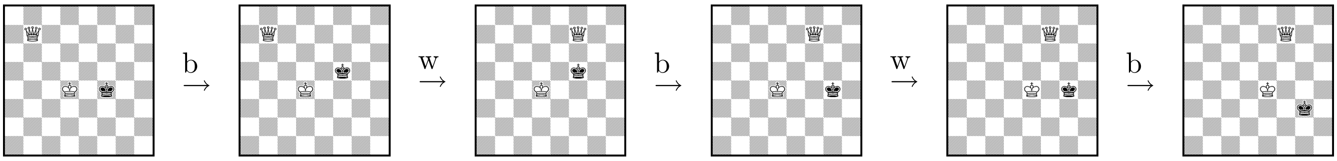
RESULTS

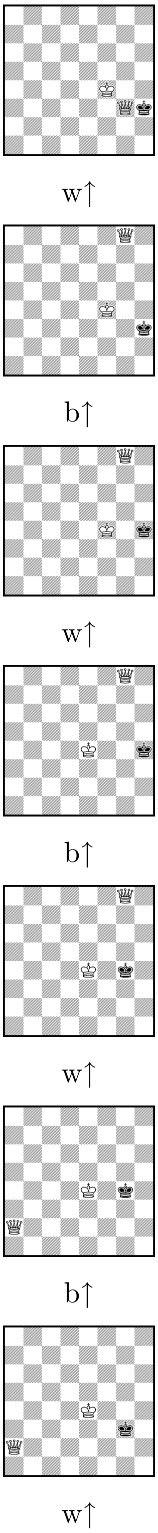
This verification technique has been used to construct golden reference endgame databases for all pawnless endgames with at most four pieces. Here is a sample results line for positions where White has a King and Queen, Black has only a King, and it is White to move:

Pieces	max	%win	#win	bdd	#legal	bdd
KQ_K	10	ALL	144508	19%	144508	1%

In this endgame all legal positions with White to move are won, and there are 144,508 of them in total. From the worst possible starting position White requires 10 moves to checkmate with perfect play, and an example of this is shown around the edge of this poster. Finally the compression ratio of number of BDD nodes divided by number of positions is 1% for the legal positions and 19% for the sum of all the winning sets.







For more information visit `http://www.gilith.com/chess/endgames`